On écrit $\boxed{P = \alpha_0(\lambda) P_0 + \alpha^*(\lambda) P^*}$

- $P_0 = ASR$
- $\alpha^*(\lambda) = \frac{1}{\lambda}$
- $\alpha_0(\lambda) = v - \frac{1}{\lambda} \tilde{\mu}' \Omega^{-1} e$
- $P^* = (\mathrm{diag}(P_0))^{-1} \Omega^{-1} \tilde{\mu}$

Théorème de séparation en 2 fonds :

Tout portefeuille efficient est la combinaison linéaire entre P_0 (l'ASR) et P^* qui ne contient que des actifs risqués.

Sharpe
$$S(a) = \frac{E[V_1(a_0,a) - v(1+r)]}{\sqrt{Var[V_1(a_0,a)]}} = \frac{w_a' \tilde{\mu}}{(w_a' \Omega w_a)^{1/2}}$$

$$S = \frac{1}{\lambda} \frac{\tilde{\mu}' \Omega^{-1} \tilde{\mu}}{\sigma} = (\tilde{\mu}' \Omega^{-1} \tilde{\mu})^{1/2} = s$$

CAPM

- on note $z = y - (1+r)e$ vecteur rentabilité net

$$z(\alpha^*) = \frac{\tilde{\mu}' \Omega^{-1} z}{\tilde{\mu}' \Omega^{-1} e} \quad \text{pour un ptf efficient}$$

on a $z = \beta z(\alpha^*_m) + \varepsilon$

$\beta = \frac{Cov(z, z(\alpha^*_m))}{Var(z(\alpha^*_m))}$. $E[\varepsilon] = 0$. $Cov[\varepsilon, z(\alpha^*_m)] = 0$

Et : $E[z_i] = \beta_i E[z(\alpha^*_m)]$

$Var[z_i] = \beta_i^2 Var[z(\alpha^*_m)] + \sigma_i^2$ avec $\sigma_i^2 = E[\varepsilon_i^2]$

Risque idiosyncratique

On prend a un ptf tq : $z(a) = w_a' z = \sum_i w_{a,i} z_i = \sum_i w_{a,i}(\beta_i z(\alpha^*_m) + \varepsilon_i)$

on pose $\to \beta(a) = \sum_i w_{a,i} \beta_i \quad \varepsilon(a) = \sum_i w_{a,i} \varepsilon_i = w_a' \varepsilon$

$\hookrightarrow E[z(a)] = \beta(a) E[z(\alpha^*_m)]$

$\hookrightarrow V[z(a)] = \beta(a)^2 Var[z(\alpha^*_m)] + Var[w_a' \varepsilon]$

$= \beta(a)^2 Var[z(\alpha^*_m)] + w_a' Var[\varepsilon] w_a$

on pose $\bar{w}_a = \frac{w_a}{\underbrace{w_a e}_{1}}$

$V[z(a)] = \beta(a)^2 Var[z(\alpha^*_m)] + \bar{w}_a' Var[\varepsilon] \bar{w}_a$